

2013 Combinatorics Workshop
(2013 조합론 학술대회)

NIMS, Daejeon, Korea
August 7-9, 2013

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2013 Combinatorics Workshop (CW2013)

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Timetable

- August 7 (Wednesday)
 - 13h30 – Registration
 - 14h20 – 14h30 Opening Ceremony
 - 14h30 – 18h00 Session 7A
 - ◊ 14h30 – 15h30 7A-1 Keynote Lecture I
 - ◊ 15h45 – 16h45 7A-2 Talk I
 - ◊ 17h00 – 18h00 7A-3 Talk II
- August 8 (Thursday)
 - 10h00 – 12h30 Session 8A
 - ◊ 10h00 – 11h00 8A-1 Talk III
 - ◊ 11h30 – 12h30 8A-2 Talk IV
 - 12h30 – 14h30 Lunch
 - 14h30 – 18h00 Session 8B
 - ◊ 14h30 – 15h30 8B-1 Keynote Lecture II
 - ◊ 15h45 – 16h45 8B-2 Talk V
 - ◊ 17h00 – 18h00 8B-3 Talk VI
 - 18h00 – 21h00 Banquet
- August 9 (Friday)
 - 10h00 – 12h30 Session 9A
 - ◊ 10h00 – 11h00 8A-1 Talk VII
 - ◊ 11h30 – 12h30 8A-2 Talk VIII
 - 12h30 – 14h30 Lunch
 - 14h30 – 18h00 Session 9B
 - ◊ 14h30 – 15h30 9B-1 Keynote Lecture III
 - ◊ 15h45 – 16h45 9B-2 Talk IX
 - ◊ 17h00 – 18h00 9B-3 Talk X
 - 18h00 – Closing Ceremony

General Information

- NIMS Wireless Internet Access
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7A-1 Two enumerative tidbits

Richard P. Stanley, *MIT, USA*

Abstract

We discuss two unrelated results in enumerative combinatorics.

- (i) Smith normal form of a matrix related to Young diagrams. We generalize a result of Carlitz, Roselle, and Scoville on a combinatorial matrix of determinant one by introducing additional parameters and computing the Smith normal form of the resulting matrix.
- (ii) A distributive lattice associated with three-term arithmetic progressions (with Fu Liu). We prove two conjectures of Noam Elkies related to arithmetic progressions of length three by showing a connection with a distributive lattice of certain semistandard Young tableaux.

**7A-2 The magical connection between balanced and well-poised series,
both hypergeometric and basic hypergeometric**

Richard A. Askey, *University of Wisconsin-Madison, USA*

Abstract

One way to start the development of hypergeometric series is with the binomial theorem, and for basic hypergeometric series with the q -binomial theorem. There are two natural first steps one can take.

$$\begin{aligned}(1-x)^a(1-x)^b &= (1-x)^{a+b} \\ (1-x)^a(1+x)^a &= (1-x^2)^a\end{aligned}$$

When the binomial theorem is used on each series and the coefficients of x^n are equated, the first identity is an instance of what will become balanced series when more parameters are introduced, and the second is an instance of well-poised series, and later very well-poised series. At this stage the two identities have nothing to do with each other. At a higher level the two chains of identities become related, Whipple's formula for hypergeometric series and Watson's extension for basic hypergeometric series. Other connections will be mentioned, and two different extensions beyond this level will be described. One, which is due to George Andrews, contains variants of the Rogers-Ramanujan identities. The other has a mysterious twist which I do not understand.

7A-3 Enumeration of Schröder families by type

Sangwook Kim, *Chonnam National University, Korea*

Abstract

Schröder paths, sparse noncrossing partitions, and partial horizontal strips are three classes of Schröder objects which carry a notion of type. We provide type-preserving bijections among these objects and an explicit formula which enumerates these objects according to type and length. We also define a notion of connectivity for these objects and discuss an analogous formula which counts connected objects by type. This is joint work with Suhyung An and Sen-Peng Eu.

8A-1 Zeta functions of adjacency algebras induced by graphs

Mitsugu Hirasaka, *Pusan National University, Korea*

Abstract

Let Γ denote a finite digraph and R_Γ denote the subring generated by the adjacency matrix of Γ . In this talk we focus on the number a_n of ideals of R_Γ with index n , and show a way to find the formal Dirichlet series $\sum_{n \geq 1} a_n n^{-s}$ when Γ can be a relation of an association scheme of prime order. This is a joint work with Akihide Hanaki.

8A-2 Jacobi-Stirling numbers and Jacobi-Stirling permutations

Jiang Zeng, *Université Claude Bernard Lyon 1, France*

Abstract

The Jacobi-Stirling numbers were discovered as a result of a problem involving the spectral theory of powers of the classical second order Jacobi differential expression. They are refinements of the Legendre-Stirling numbers and generalize the Stirling numbers and central factorial numbers. In this talk I will report on the recent work about the combinatorics of these numbers. Moreover, the diagonal generating functions of Jacobi-Stirling numbers are rational functions, of which the numerators are the enumerative polynomials of the Jacobi-Stirling permutations.

8B-1 Polynomial sequences of binomial type

Richard P. Stanley, *MIT, USA*

Abstract

A sequence $p_0(n), p_1(n), \dots$ of complex polynomials is of *binomial type* if $p_0(n) = 1$ and

$$\sum_{k \geq 0} p_k(n) \frac{x^k}{k!} = \left(\sum_{k \geq 0} p_k(1) \frac{x^k}{k!} \right)^n .$$

Such polynomials play a fundamental role in the theory of operator calculus developed by G.-C. Rota and his collaborators. After briefly reviewing this theory, we will focus on examples of such polynomials. In particular, we discuss recent work of J. Schneider related to placing figures on tori. An application is given to chromatic polynomials of toroidal grid graphs.

8B-2 Combinatorics of $2D$ -Hermite polynomials

Mourad E. H. Ismail, *University of Central Florida, USA*

Abstract

We discuss the combinatorics and generating functions of the $2D$ -Hermite polynomials. This is partly based on joint work with Plamen Simeonov.

8B-3 Introduction to the a-number of graphs and hypergraphs

Suyoung Choi, *Ajou University, Korea*

Abstract

Recently, I and my colleague Park have introduced new combinatorial invariants, called the a-number, of any finite simple graph, which arise in toric topology. Interestingly, for specific families of the graph, our invariants are deeply related to well-known combinatorial sequences such as the Catalan numbers and Euler zigzag numbers. In this talk, I introduce several further works on this topic, and I will discuss about the analogue of the invariant for hypergraphs.

9A-1 Littlewood-Richardson numbers of Schur's S - and P -Functions

Soojin Cho, *Ajou University, Korea*

Abstract

Littlewood-Richardson numbers(LR-numbers) are structure constants of Schur's S -functions. Many combinatorial models for LR-numbers of Schur functions are known and they are very well understood. We review known combinatorial rules to calculate LR-numbers and interesting properties of LR-numbers including symmetries and factorization theorem. We then introduce eight useful reduction formulae deleting one or two rows (columns) of each partition. As an application, we prove that if the LR-number is 1 and each partition has distinct parts, then one of two types of our reduction formulae is always applicable and hence we have an algorithm to test if the LR-number is 1.

There do not exist so many combinatorial models of LR-numbers for Schur's P -functions. We introduce a new LR-rule for Schur's P -functions and some properties of them.

9A-2 Signed counting of Euler numbers

Sen-Peng Eu, *National University of Kaohsiung, Taiwan*

Abstract

Euler numbers count several important classes of permutations, among them the alternating permutations and the simsun permutations. In this talk we introduce some new results on the signed counting of these permutations.

9B-1 Valid orderings of hyperplane arrangements

Richard P. Stanley, *MIT, USA*

Abstract

Let \mathcal{A} be a finite set of hyperplanes in \mathbb{R}^n . Let L be a sufficiently generic directed line in \mathbb{R}^n . Then L intersects the hyperplanes in \mathcal{A} in a certain order, called a *valid ordering* of \mathcal{A} . We will discuss connections between valid orderings and such topics as line shellings of polytopes, the Dilworth truncation of a matroid, and a generalization of chromatic polynomials. Some knowledge of matroid theory will be useful but not essential for understanding this lecture.

9B-2 Reflection factorizations of Singer cycles

Dennis Stanton, *University of Minnesota, USA*

Abstract

The number of factorizations of an n -cycle in S_n into $n - 1$ transpositions is n^{n-2} . We consider a version of this theorem when $GL_n(F_q)$ replaces S_n , the Singer cycle replaces an n -cycle, and reflections replace transpositions. We give explicit enumeration formulas for this question, and also longer factorizations. The answers involve a mixture of binomial and q -binomial coefficients. Character techniques are used, no bijective proofs are known. This is joint work with Joel Lewis and Vic Reiner.

9B-3 Moments of Askey-Wilson polynomials

Jang Soo Kim, *KIAS, Korea*

Abstract

The Askey-Wilson polynomials are the most general orthogonal polynomials among those classified by the Askey scheme. These are orthogonal polynomials in one variable with 5 parameters. In this talk I will talk about 3 combinatorial methods to study the n -th moment of the Askey-Wilson polynomials. The first method is Viennot's theory of weighted Motzkin paths. The second method uses staircase tableaux introduced by Corteel and Williams. The third method is a modification of an idea of Ismail, Stanton, and Viennot on matchings and q -Hermite polynomials. Using the third method we express the n -th moment as a fraction of two generating functions for certain matchings and obtain a new formula for the moment. This is joint work with Dennis Stanton.

Registered participants

Students are denoted by ϵ .

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- (2) Richard A. Askey, *University of Wisconsin-Madison, USA*
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- (5) Soojin Cho (조수진), *Ajou University*
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- (18) Masao Ishikawa (石川 雅雄), *University of the Ryukyus, Japan*
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